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THE LE DUC VELOCITY FORMULA.

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In connection with the modified Sarrau formulæ proposed by Mr. Patterson in the preceding article, it may be interesting to consider another semi-empirical formula which has been in use in the Bureau of Ordnance and at the Indian Head Proving Ground for several years past, and which gives remarkably accurate results.

This formula, which was originally found in a series of articles in the *Revue d'Artillerie*,* where it was designated as Captain Le Duc's semi-empirical formula, assumes the space velocity curve of a projectile in the gun to be a hyperbola whose equation is

$$v = \frac{au}{b+u},$$

a and b being constants, u the travel of the projectile and v its velocity.

It will readily be seen that a is the value assumed by v when u becomes infinite, and that by varying the values of a and b the shape of the curve may be made either very flat or very much rounded so that it can represent the effect of either slow or quick powders.

In order to determine the value of a , we have to consider that it is the velocity that the projectile would attain in a gun of infinite length, and that in such a case the muzzle energy of the projectile would equal the whole work the powder charge was capable of doing.

If p is the pressure and v the volume of unit weight of a gas, and if n is the ratio of the specific heats of the gas, pv^n remains

* *Théorie des affûts à Déformation*. By J. Challéat, Capitaine d'artillerie. *Revue d'artillerie*, 1904 and 1905.

constant in adiabatic expansion, and the work done in expanding adiabatically from volume v_1 to volume v_2 is $\int_{v_1}^{v_2} p dv$.

Therefore we have

$$pv^n = k.$$

$$\text{Work} \int_{v_1}^{v_2} p dv = \int_{v_1}^{v_2} \frac{k}{v^n} dv = \frac{k}{n-1} \left[\frac{1}{v_1^{n-1}} - \frac{1}{v_2^{n-1}} \right],$$

or, if $v_2 = \infty$,

$$\text{Work} \int_{v_1}^{\infty} p dv = \frac{k}{n-1} \cdot \frac{1}{v_1^{n-1}}.$$

Hence, if we call the work done by one pound of gas in expanding from unit density (27.68 cu. in. per pound) to infinity the potential (E), we have

$$E = \frac{k}{n-1} \cdot \frac{1}{27 \cdot 68^{n-1}}.$$

And if the expansion is from any other density (Δ), to infinity, since $v_1 = \frac{27 \cdot 68}{\Delta}$, the work done will be

$$\frac{k}{n-1} \cdot \frac{1}{v_1^{n-1}} = \frac{k}{n-1} \cdot \frac{\Delta^{n-1}}{27 \cdot 68^{n-1}} = E \Delta^{n-1}.$$

This work, multiplied by the weight of the powder charge (w), must equal the energy of the projectile $\left(\frac{pa^2}{2g} \right)$, and so we get

$$\frac{pa^2}{2g} = E w \Delta^{n-1},$$

$$a^2 = 2gE \cdot \frac{w}{p} \cdot \Delta^{n-1},$$

$$a = \sqrt{2gE} \left(\frac{w}{p} \right)^{\frac{1}{2}} \Delta^{\frac{n-1}{2}}.$$

The value of $\sqrt{2gE}$ in foot second units, giving E its actual value of 653 foot-tons per pound, works out to be 9706, but this would not serve our purpose, which is to find a value in accord with experimental results. A large part of the energy of the powder is wasted in overcoming frictional resistances and in giving velocity to the charge itself; so that the empirical value of a is very much less than its value calculated as above. Actually the value 6857 f. s. suits our powder very well.

As for the value of $\frac{n-1}{2}$, this may be taken as $\frac{1}{2}$.

Since the other constant (b) is twice the travel to the point of maximum pressure, it must depend first of all upon the quickness of the powder, or, with any particular kind of powder, upon the dimensions of the grain. Then, as Sarrau has shown, it must be proportional to the initial air space, and, finally, it must be inversely proportional to some power of the chamber volume and also to some power of the projectile weight, since increase of either of these diminishes the distance to the point of maximum pressure.

Therefore, if s =chamber volume, δ =density of powder, Δ =density of loading, p =weight of projectile and β is a powder constant, we have

$$b = \beta \frac{s \left(1 - \frac{\Delta}{\delta}\right)}{s^x p^y}.$$

And, taking $x = \frac{5}{8}$, $y = \frac{3}{8}$ for empirical values, and putting $\delta = 1.56$, this becomes

$$b = \beta (1 - .64\Delta) \left(\frac{s}{p}\right)^{\frac{3}{8}}.$$

To get the value of the pressure we differentiate the velocity formula, thus getting the acceleration of the projectile, and this, multiplied by the mass of the projectile and divided by the cross-sectional area of the bore (ω), gives the *effective* pressure per unit area on the base of the shell.

$$v = \frac{au}{b+u},$$

$$\frac{dv}{dt} = \frac{ab}{(b+u)^2} \frac{du}{dt} = \frac{abv}{(b+u)^2} = \frac{a^2bu}{(b+u)^3},$$

$$P = \frac{p}{g\omega} \frac{dv}{dt} = \frac{pa^2b}{g\omega} \frac{u}{(b+u)^3}.$$

And the maximum pressure is found by putting $u = \frac{b}{2}$ in this, thus getting

$$P_{max} = \frac{4}{27} \frac{p}{g\omega} \frac{a^2}{b}.$$

To recapitulate, the Le Duc formulæ, in our units, are as follows:

$$a = a \left(\frac{\varpi}{p} \right)^{\frac{1}{2}} \Delta^{\frac{1}{2}}. \quad (1)$$

$$b = \beta (1 - .64\Delta) \left(\frac{s}{p} \right)^{\frac{3}{8}}. \quad (2)$$

$$v = \frac{au}{b+u}. \quad (3)$$

$$P = \frac{p}{g} \frac{a^2 b}{\omega} \frac{u}{(b+u)^3}. \quad (4)$$

$$P_{max} = \frac{4}{27} \frac{p}{g\omega} \cdot \frac{a^2}{b}. \quad (5)$$

In all of which

ϖ = weight of charge in pounds.

p = weight of projectile in pounds.

Δ = density of loading.

s = chamber volume in cubic inches.

a = a constant whose value depends only on the chemical characteristics of the powder and may be taken as 6857 for the present U. S. Navy powder.

β = a constant whose value (for any given kind of powder) depends only on the form and dimensions of the grain.

v = velocity of projectile in f. s.

u = travel of projectile in bore of gun in feet.

ω = cross-section of bore in square inches.

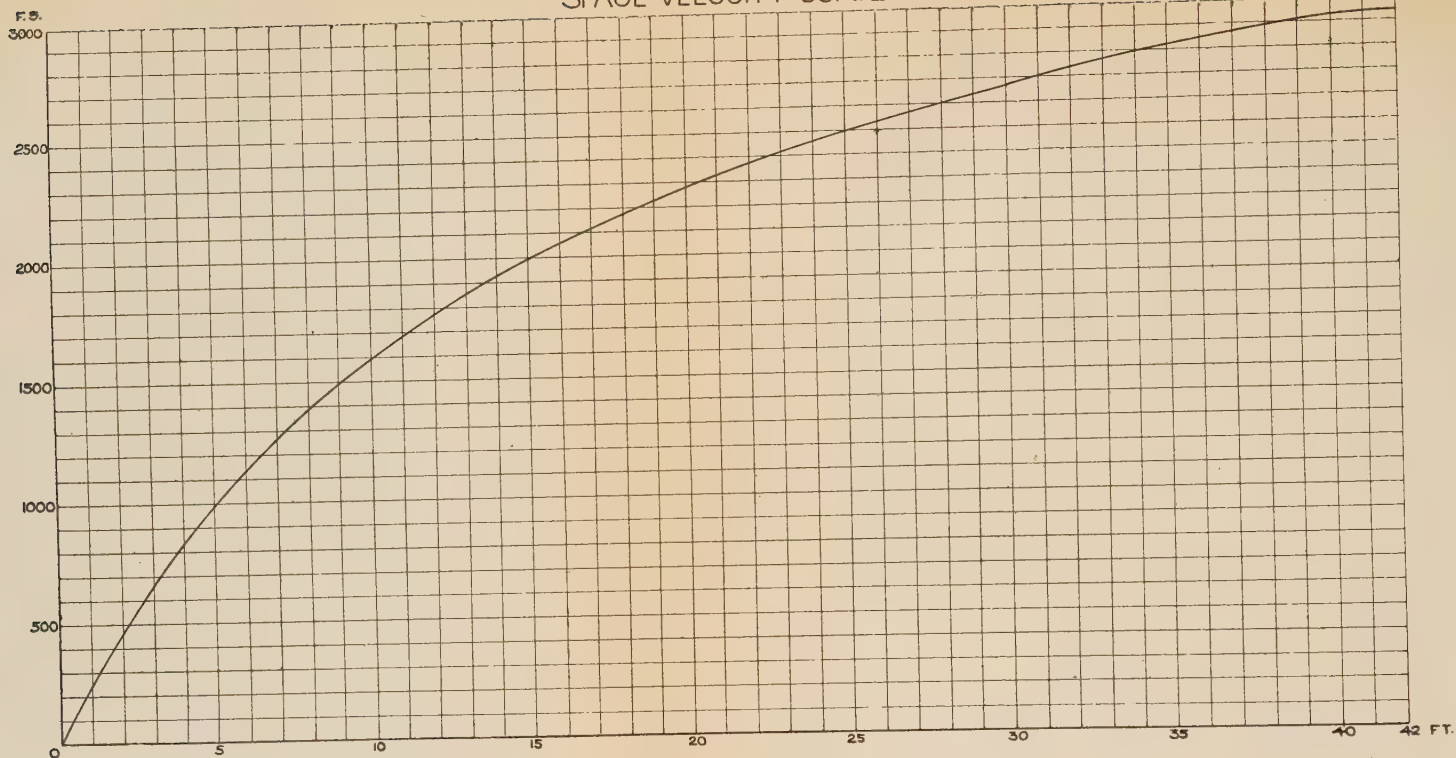
g = acceleration of gravity in f. s. s.

P = effective pressure on base of projectile in pounds per square inch.

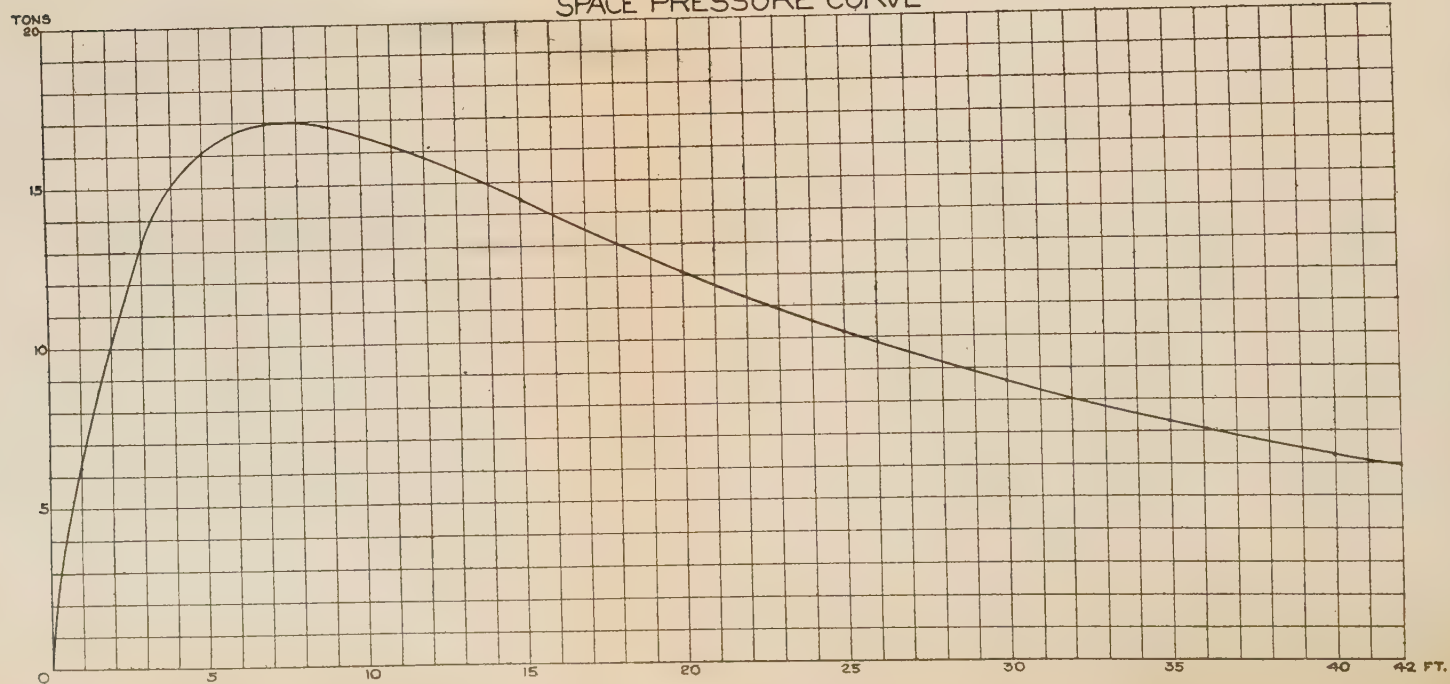
P_{max} = maximum value of P .

The commonest use of these formulæ is in the case where the velocity given by a certain charge in a given gun has been measured. Knowing ϖ , p , Δ and a , the value of a is calculated from (1). Then, knowing the value of v corresponding to the total travel in the bore, b is found from (3), and after that the values of v and P at any point can be calculated from (4) and the maximum value of P from (5). Furthermore, b having been found, β can be calculated from (2) and then new values of a and b can be found corresponding to any desired values of ϖ , p and Δ , and the velocities and pressures given under the new conditions found from (3), (4) and (5).

12-IN. 50-CAL. GUN
SPACE-VELOCITY CURVE



12-IN. 50-CAL. GUN
SPACE PRESSURE CURVE



The diagrams show space-velocity and space-pressure curves in the 50-caliber 12-inch gun ($\omega=328$, $p=870$, $v=2950$ f. s. for $u=42$ feet) calculated in this manner. The value of b is 15.2 feet and the maximum pressure 16.96 tons.

It must be remembered that the pressures thus calculated are the *effective* pressures, so that to get the actual gas pressure at the base of the projectile it is necessary to add from 1 to 2 tons per square inch during the first part of the travel and from .5 to 1 ton per square inch near the muzzle. Moreover, the pressure at the bottom of the chamber, where the gauges measure it, is supposed to be from 15% to 20% greater than that at the base of the projectile.

The following results will show how accurately these formulæ give muzzle velocities for different conditions of loading in one gun and for different lengths of gun using the same charge:

6-inch Brown Wire-wound gun (using a value of β calculated from one round):

ω	V meas.	V cal.
32.60	1905	1812
49.00	2500	2466
58.33	2865	2847
64.33	3100	3101
65.33	3140	3144
66.33	3180	3187
67.33	3220	3230
68.33	3260	3274
68.83	3286	3296

Three 6-inch guns of different lengths (using β as calculated from one gun):

	ω	V meas.	V cal.
30-caliber gun	24.3	2210	2209
35-caliber gun	24.3	2320	2318
40-caliber gun	24.3	2400	2403

Two 8-inch cut-off guns (using β calculated from a whole gun):

ω	Gun with 74" cut-off.		Gun with 128" cut-off.	
	V meas.	V cal.	V meas.	V cal.
35.5	1605	1606	1404	1427
40.5	1752	1751	1538	1560
43.0	1827	1822	1609	1625
45.5	1897	1892	1683	1690
48.0	1967	1962	1758	1755

These foregoing figures prove that the formula gives the effective pressure near the muzzle with great accuracy, since otherwise it would not reproduce so closely the results obtained with different lengths of bore. To show how calculated maximum pressures compare with gauge pressures the following example will suffice:

6-inch, 40-caliber gun:

$\bar{\omega}$	V	β (cal.)	P_{max} (for $\beta=2.41$) tons	P (gauge) tons
22.5	2395	2.396	15.7	17.5
24.0	2498	2.409	17.4	19.6
24.4	2516	2.459	17.9	20.1
27.0	2694	2.476	21.2	24.5
29.0	2848	2.396	24.7	26.3
30.2	2940	2.346	26.0	29.0

2.41 mean value

When we come to the crucial test, however, and try to ascertain the results that a certain powder will give in a gun by using the value of β calculated from the results obtained in another gun (of different caliber), we do not meet with complete success, as the following examples will show.

10-inch, 30-caliber and 13-inch, 35-caliber guns (using β calculated from results given by same powder in an 8-inch, 45-caliber gun) :

10" gun.			13" gun.		
$\bar{\omega}$	V meas.	V cal.	$\bar{\omega}$	V meas.	V cal.
80	1656	1655	180	1911	1858
95	1925	1857	190	1988	1923
99	1986	1910	200	2078	1988
109	2149	2041			

Again, the mean value of β calculated from five rounds (using different weights of charge) in the 12-inch 45-caliber gun was 5.98, while for the same powder in the 10-inch 40-caliber gun the mean value from the rounds (with different weights of charge) was 6.36.

However, while these results show the formula not to be perfect, it may at least be said to give fairly accurate results in even the worst cases, and its simplicity certainly commends it greatly.